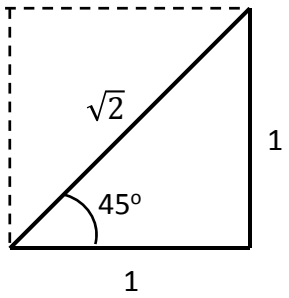
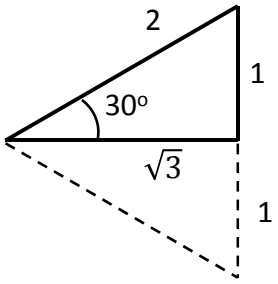
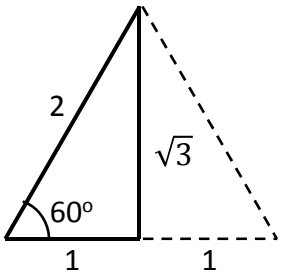
Recreate the Unit Circle:

Question: Where do the values of the trigonometric functions for the angles on the unit circle come from?

Answer: They come from two "special" right triangles: the isosceles right triangle and the 30-60-90 right triangle.







The isosceles right triangle gives the values for 45 degrees.

Take a square of side 1 and draw a diagonal.

The hypotenuse has length $\sqrt{2}$. So

$$\sin 45° = \frac{\sqrt{2}}{2} \quad \cos 45° = \frac{\sqrt{2}}{2}$$

$$\tan 45° = 1$$

The 30-60-90 right triangle gives the values for 30 degrees and for 60 degrees.

Take an equilateral triangle of side 2 and draw a perpendicular bisector.

The bisector has length $\sqrt{3}$. So

$$\sin 30° = \frac{1}{2} \quad \cos 30° = \frac{\sqrt{3}}{2} \quad \tan 30° = \frac{\sqrt{3}}{3}$$

and

$$\sin 60° = \frac{\sqrt{3}}{2} \quad \cos 60° = \frac{1}{2} \quad \tan 60° = \sqrt{3}$$

Now build a table:

1. Write the numbers 0 – 4 in a row.
2. Write the angles 0, 30, 45, 60, and 90 degrees beneath them.
3. For the values of sine, take the square roots of the numbers in the top row and
4. Divide by 2:
5. For the values of cosine, start with the last number for sine and write the values backwards
6. $\tan\theta$ is just $\sin\theta$ / $\cos\theta$

	0	1	2	3	4
	0°	30°	45°	60°	90°
	$\sqrt{0}$	$\sqrt{1}$	$\sqrt{2}$	$\sqrt{3}$	$\sqrt{4}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef

To complete the values for the unit circle, we need two more things:

1. The signs of the trigonometric functions in different quadrants
2. The angles in different quadrants that have 30, 45, and 60 degrees as <u>reference angles</u>

For #1, remember "All Students Take Calculus"

STUDENTS Quadrant 2	**A**LL Quadrant 1
S stands for sine Only sine is positive $\sin\theta > 0$ $\cos\theta < 0$ $\tan\theta < 0$	ALL are positive $\sin\theta > 0$ $\cos\theta > 0$ $\tan\theta > 0$
$\sin\theta < 0$ $\cos\theta < 0$ $\tan\theta > 0$ T stands for tangent Only tangent is positive	$\sin\theta < 0$ $\cos\theta > 0$ $\tan\theta < 0$ C stands for cosine Only cosine is positive
Quadrant 3 **T**AKE	Quadrant 4 **C**ALCULUS

Now we can make the tables for the other quadrants:

Quadrant 2:

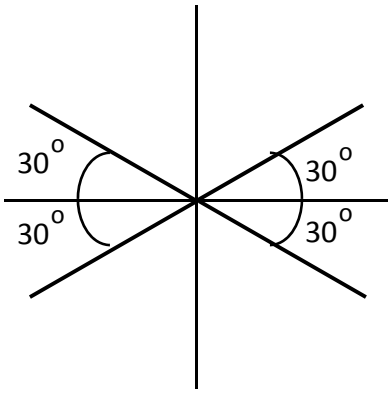
Θ	120°	135°	150°	180°
Ref Θ	60°	45°	30°	0°
$\sin\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos\theta$	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan\theta$	$-\sqrt{3}$	1	$-\frac{\sqrt{3}}{3}$	0

Quadrant 3:

Θ	210°	225°	240°	270°
Ref Θ	30°	45°	60°	90°
$\sin\theta$	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\cos\theta$	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	0
$\tan\theta$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef

For #2, use reference angles:

Every angle in quadrants 2, 3, and 4 have a corresponding <u>reference angle</u> in quadrant 1 whose trigonometric values are the same, except for the sign.



The figure shows that 30° is the reference angle for 150°, 210°, and 330°.

Similarly, 60° is the reference angle for 120°, 240°, and 300°.

45° is the reference angle for 135°, 225°, and 315°.

Quadrant 4:

Θ	300°	315°	330°	360°
Ref Θ	60°	45°	30°	0°
$\sin\theta$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
$\cos\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan\theta$	$-\sqrt{3}$	1	$-\frac{\sqrt{3}}{3}$	0