I. CLEARLY AND COMPLETELY STATE

1. The definition of continuity of a function $f(x)$ at $x = a$ and how to use this definition in a problem.

2. The formal definition of $f'(x)$, the derivative of a function $f(x)$.

3. The definition of a critical value of a function and how to tell whether the function has a local minimum, a local maximum, or neither at each of its critical values.

4. How to tell where a function is increasing and where it is decreasing.

5. How to tell where a function is concave upward or concave downward.

6. How to locate possible inflection points of a function and to determine whether a function really has an inflection point at each possible inflection point.

7. The Fundamental Theorem of Calculus. Why is this theorem so important?

II REVIEW PROBLEMS

1. Evaluate each limit:

(a) $\lim\limits_{x \to 4} \dfrac{x-4}{x^2 - x - 12}$

(b) $\lim\limits_{x \to \infty} \dfrac{3x-2}{9x-7}$

(c) $\lim\limits_{x \to -\infty} \dfrac{2x^3}{x^2+1}$

(d) $\lim\limits_{x \to 3^-} \dfrac{2x}{9-x^2}$

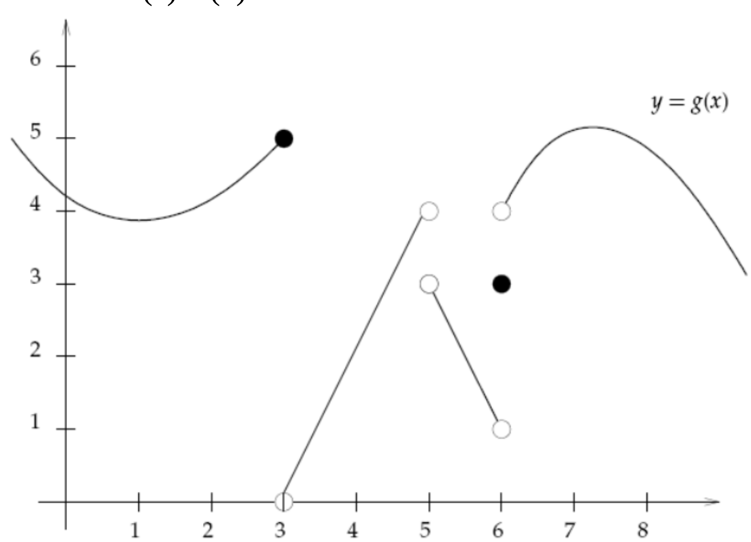
Use the graph of the function g to evaluate the limits for (e) – (h) below.

(e) $\lim\limits_{x \to 5^-} g(x)$

(f) $\lim\limits_{h \to 0} \dfrac{g(4+h) - g(4)}{h}$

(g) $\lim\limits_{x \to 6^+} \dfrac{x-5}{g(x)}$

(h) $\lim\limits_{x \to 6} g(x)$

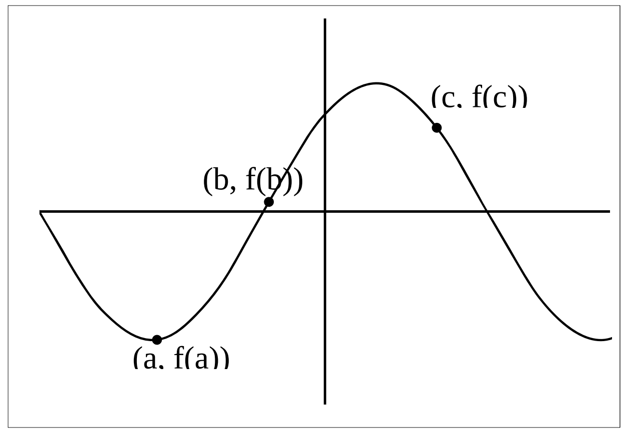


2. List all vertical and horizontal asymptotes of $y = \dfrac{2x}{x^2+5x-6}$.

3. Let $f(x) = \begin{cases} x^2 - 8 & \text{if} \quad x \le a \\ \dfrac{x^2 - a^2}{x - a} & \text{if} \quad x > a \end{cases}$

   Find all values of *a* so that *f* is continuous for all *x*.

4. Use the graph of $f(x)$, given to the right, to decide if each of the following is positive, negative or zero.



(a)	$f(a)$	(b)	$f(b)$	(c)	$f(c)$
(d)	$f'(a)$	(e)	$f'(b)$	(f)	$f'(c)$
(g)	$f''(a)$	(h)	$f''(b)$	(i)	$f''(c)$

5. Find the equation of the line tangent to the curve $y = \sqrt{9-4x}$ when $x = -4$ on the curve.

6. Find $\dfrac{dy}{dx}$ for each of the following:

(a) $y = (2x^3 + 5x^2 - 6x - 4)^5$

(b) $y = \tan^3(5x)$

(c) $y = \sqrt{\dfrac{x}{3x+1}}$

(d) $y = (x^2+4)^2(2x^3-1)^3$

(e) $4x^2y + x^2 - 6y^4 = 5$

(f) $y = e^{2x}\sin x$

(g) $y = \ln\dfrac{(5x+2)^4}{\sqrt{x^2+5}}$

(h) $y = \arctan(x^3)$

7. Use the first and second derivatives to determine all extrema and inflection points for the curve $y = x^5 - 30x^3 + 6$.

8. Discuss the concavity of the function $y = x^3 - 6x^2 + 9x$.

9. Analytically determine the absolute minimum and the absolute maximum of $f(x) = x^3 - 2x^2$ on $[-1,1]$.

10. A power company needs to lay a cable from point A on one bank of an 800-foot wide, straight river to point B on the opposite bank 1600 feet downstream. The power company will choose a point C on the same side of the river as B, and will lay a cable underwater from A to C and on land from C to B. It costs \$3 a foot to lay the cable on land and \$5 a foot to lay it underwater. Where should point C be chosen to minimize the total cost and what is the minimum cost?

11. A closed box with square base is to be built to house an ant colony. The bottom of the box and all four sides will be made of material costing \$1 per square foot, and the top is to be constructed of glass costing \$5 per square foot. What are the dimensions of the box of greatest volume that can be constructed for \$72?

12. A weather balloon is rising vertically at the rate of 10 ft/s. An observer is standing on the ground 300 ft horizontally from the point where the balloon was released.
    (a) At what rate is the distance between the observer and the balloon changing when the balloon is 400 ft high?
    (b) At what rate is the angle of elevation of the observer's line of sight to the balloon changing when the balloon is 400 feet high?

13. Evaluate each definite or indefinite integral:

(a) $\int_{-1}^{2} (3x^2 - x + 2)\, dx$ (b) $\int \frac{2x^2 + 5x}{x^3}\, dx$ (c) $\int \left(2\cos t - 5\sec^2 t\right) dt$

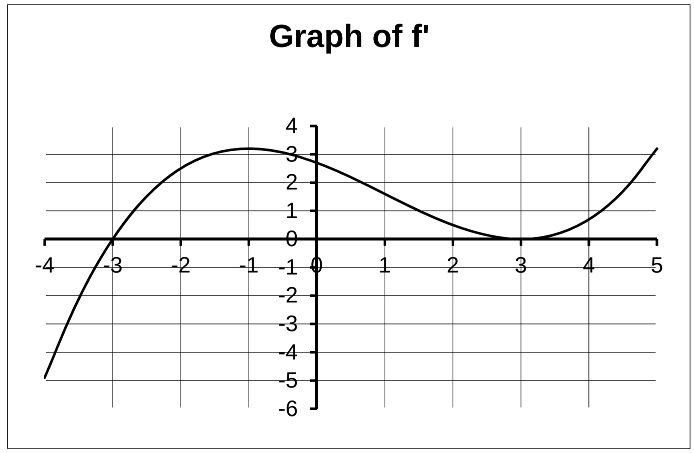
(d) $\int_0^4 (2v+5)(3v-1)\, dv$ (e) $\int \frac{\sin(2x)}{\sin x}\, dx$ (f) $\int_0^{3\pi/2} |\sin x|\, dx$

14. An object moves according to the equation $s = t(2t-1)^3$, where s is the position of the object in feet and t is time in seconds. Find the acceleration when $t = 1$.

15. Use Newton's Method to approximate the root of $\cos x + x = 2$ to four decimal places.

16. For the functions $f(x) = \frac{1}{1-x^2}$ and $g(x) = \frac{x^2}{1-x^2}$.
    (a) Show that these functions have the same derivative.
    (b) What does this imply about the relationship between these functions? That is, are they the <u>same</u> function? If not, in what way do they differ? You may find it helpful to graph the functions.

17. The graph of the <u>derivative</u> of a certain function *f* appears below.



(a) Suppose $f(1)=5$. Find an equation of the line tangent to the graph of *f* at (1,5).

(b) Suppose $f(-1)=-2$. Could $f(3)=-6$? Why or why not?

(c) Estimate $f''(-3)$.

(d) At which values of *x* does $f(x)$ have points of inflection?

(e) At which value of *x* in the interval $[-4,5]$ does $f(x)$ achieve its largest value? Its smallest value?

18. Two curves are said to cross at right angles if their tangent lines are perpendicular at the crossing point. The technical word for "crossing at right angles" is *orthogonal.* Show that the curves $y=\sin 2x$ and $y=-\sin\left(x/2\right)$ are orthogonal at the origin. Draw both graphs and both tangent lines in a square viewing window to confirm your results.

19. Explain the difference between the <u>average</u> rate of change in a function and the <u>instantaneous</u> rate of change. What's the difference in the way you compute these quantities? Illustrate by considering the average rate of change of the function $f(t)=t^3+5t$ on the interval $[1.9, 2.1]$ and the instantaneous rate of change of the same function at $t=2$.

20. Some values of a function *f* and its derivative f' are tabulated below:

x	-3	-2	-1	0	1	2	3
$f(x)$	4	-1	1	3	-2	2	5
$f'(x)$	0	1	2	-1	3	-2	-3

(a) Let $g(x)=2f(x+3)$. Evaluate $g'(-2)$.

(b) Let $h(x)=\dfrac{\ln x}{f(x)}$. Is $x=1$ a critical number of *h*? Justify your answer.

(c) Let $j(x)=f(x^4)$. Is *j* increasing or decreasing at $x=-1$? Justify your answer.

(d) Suppose that *F* is an antiderivative of *f* and that $F(0)=3$. If $k(x)=\left(F(x)\right)^2$, is *k* concave up or concave down at $x=0$? Justify your answer.

21. Suppose that $N(t)$, the number of bacteria is a culture at time *t* is given by $N(t) = 25 + te^{-t/20}$, where *N* is measured in millions of bacteria and *t* is in hours.
    (a) At what time during the interval [0,100] is the number of bacteria the smallest? What is the minimum number?
    (b) At what time during the interval [0,100] is the number of bacteria the largest? What is the maximum number?
    (c) At what time during the interval [0,100] is the rate of change of the number of bacteria a minimum?

22. Find a function $y = f(x)$ with the following properties:
    (i) $f''(x) = 6x$
    (ii) Its graph passes through the point $(0,1)$ and has a horizontal tangent there.

23. The acceleration of gravity near the surface of Mars is 3.72m/sec^2. If a rock is blasted straight up from the surface with an initial velocity of 93 m/sec, how high does it go?

24. Find $\int_2^4 x \ln x \, dx$ in two ways.
    (a) Use left and right sums with $n = 50$ and average your results.
    (b) One of the three functions *F*, *G* and *H* below is an antiderivative of the function $x \ln x$. Decide which function it is and then use the Fundamental Theorem of Calculus to evaluate the given integral.
    $F(x) = x \ln x - x$, $G(x) = x^2 (2 \ln x - 1)/4$, $H(x) = (x^2 \ln x)/2$

25. Given the curve $x^2 y - 5xy^3 + 6 = 0$,
    (a) Find the slope and equation of the tangent line to this curve at $(3,1)$.
    (b) Use a linear approximation to estimate the *y*-value of a point on this curve with *x*-value 3.2.

26. Let $f(x) = x^2 + \frac{a}{x}$.
    (a) What value of *a* makes *f* have a local minimum at $x = 2$?
    (b) What value of *a* makes *f* have a point of inflection at $x = 1$?
    (c) Is there any value of *a* for which *f* can have a local maximum? Why or why not?

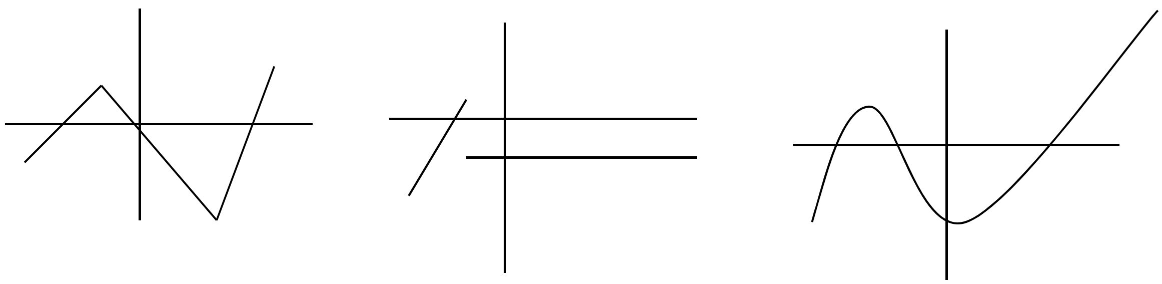
27. You are designing right circular cylindrical cans with volumes of 1000 cubic centimeters. The manufacturer of these cans will take waste into account. There is no waste in cutting the aluminum for the sides, but the tops and bottoms of radius *r* will be cut from squares that measure 2*r* cm on a side. The total amount of aluminum used up by each can will therefore be $A = 8r^2 + 2\pi rh$. Find the values of *r* and *h* which will minimize the amount of aluminum used.

28. Which of the graphs below suggest a function $y = f(x)$ that is

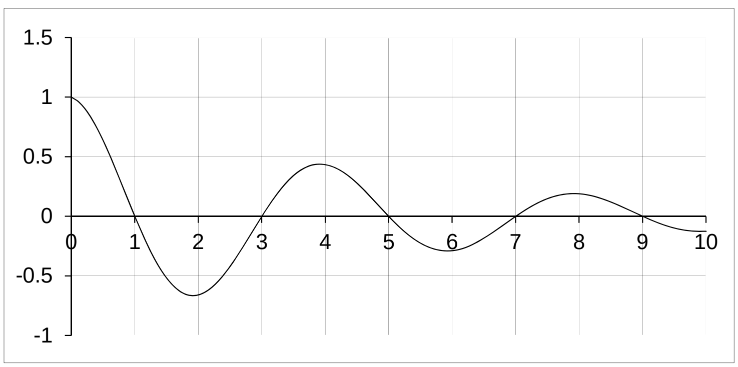
(a) continuous for all real *x*?
(b) Differentiable for all real *x*?
(c) Both (a) and (b)?
(d) Neither (a) nor (b)?

Explain in each case.

I II III



29. Let $g(x)$ be defined by $g(x) = \int_0^x f(t)dt$. The graph of *f* is shown below.



(a) Does $g(x)$ have any local maxima within the interval $[0,10]$? If so, where are they located?

(b) At what value of *x* does $g(x)$ attain its absolute minimum value on the interval $[0,10]$?

(c) On which subinterval(s) of $[0,10]$, if any, is the graph of $g(x)$ concave up? Justify your answer.

(d) Approximate the values of $g(1)$ and $g(3)$.

30. The velocity of an object moving on a horizontal line is given by v(t) = 8 - 2t (in ft/sec). Find

(a) the displacement during the time interval $0 \le t \le 6$.
(b) the total distance traveled during the time interval $0 \le t \le 6$.

REVIEW PROBLEMS - ANSWERS

1. (a) 1/7 (b) 1/3 (c) $-\infty$ (d) $+\infty$
   (e) 4 (f) 2 (g) ¼ (h) limit does not exist

2. $y=0$, $x=1$, $x=-6$

3. $a=-2,\ a=4$

4.
   (a) negative (b) positive (c) positive
   (d) zero (e) positive (f) negative
   (g) positive (h) zero (i) negative

5. $y=-\frac{2}{5}x+\frac{17}{5}$ or $2x+5y-17=0$

6.
   (a) $5(2x^3+5x^2-6x-4)^4(6x^2+10x-6)$ (b) $15\tan^2(5x)\sec^2(5x)$

   (c) $\dfrac{1}{2x^{\frac{1}{2}}(3x+1)^{\frac{3}{2}}}$ (d) $2x(x^2+4)(2x^3-1)^2(13x^3+36x-2)$

   (e) $-\dfrac{x+4xy}{2x^2-12y^3}$ (f) $e^{2x}\left(\cos x+2\sin x\right)$

   (g) $\dfrac{20}{5x+2}-\dfrac{x}{x^2+5}$ (h) $\dfrac{3x^2}{x^6+1}$

7. Rel. max at $x=-3\sqrt{2}$
   infl. pts. at $x=-3,0,3$
   Rel. min at $x=3\sqrt{2}$

8. Concave down for $x<2$
   Concave up for $x>2$

9. -3, 0

10. Go diagonally across the river to a point 600 ft. downstream, then along land for 1000 ft.
    Min. cost = $8000

11. base: 2 ft by 2 ft, height: 6 feet

12. (a) 8 ft/s (b) .012 radians/s

13. (a) 27/2 (b) $2\ln|x|-\frac{5}{x}+C$ (c) $2\sin t-5\tan t+C$
    (d) 212 (e) $2\sin x+C$ (f) 3

14. 36 ft/sec^2

15. 2.9883

16. $f\left(x\right)=g\left(x\right)+1$

17. (a) $y = 1.5x + 3.5$
(b) No, since $f'(x) > 0$ for $x > -3$, $f(x)$ is always increasing on this interval
c) $f''(-3) \approx 4$
d) at $x = -1$ and $x = 3$
e) largest value at $x = 5$, smallest value at $x = -3$

18. Show that at $x = 0$ the derivative of the first curve is the negative reciprocal of the derivative of the second curve.

19. $V_{av} = 17.01, V = 17$

20. (a) 6
(b) no, $h'(1) \neq 0$
(c) *j* is decreasing because $j'(-1) = -12 < 0$
(d) $k''(0) = 12 > 0$, so $k(x)$ is concave up

21. (a) $N(0) = 25$ million
(b) $N(20) = 32.36$ million
(c) when $t = 40$ hours

22. $f(x) = x^3 + 1$

23. 1162.5 m

24. (a) $\int_2^4$ x ln x dx = 6.704
(b) $G(4) - G(2) = 8\ln 4 - 2\ln 2 - 3 \approx 6.704$

25. (a) slope = 1/36, $x - 36y + 33 = 0$
(b) 1.00555...

26. (a) 16
(b) -1
(c) no

27. $r = 5\,\text{cm}, \quad h = \dfrac{40}{\pi}\,\text{cm}$

28. (a) I and III (b) III (c) III (d) II

29. (a) local maxima occur at $x = 1, 5, 9$
(b) at $x = 3$
(c) *g* is concave up on (2,4) and on (6,8) because $f' = g''$ is positive on those intervals
(d) $g(1) \approx .6, \quad g(3) \approx -.2$ or $-.3$

30. (a) 12 ft
(b) 20 ft